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Wakefield Focusing: New RF Technology for Electron Storage Rings

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Abstract

We propose a novel passive device that can be used in electron storage rings to provide for shorter bunches, without any changes in RF system, or the same bunch length, with fewer RF cavities. The physics of bunch shortening through wakefield focusing is analyzed. Various instabilities that could be of concern are discussed. The wakefield device, a dielectric-lined waveguide, is essentially a beam-driven monochromatic microwave generator and presents a new RF technology for electron storage rings.

The length of bunches in high-energy electron storage rings has to be minimized in order to obtain minimum beta functions and maximize the luminosity (see, e.g. [1]). This length, for a given momentum spread of the bunch, is the product of the focusing effect of the externally excited RF cavities and the normally a defocusing effect of the longitudinal broadband impedances. The latter produce bunch lengthening through either potential well distortion or coherent instabilities (turbulent bunch lengthening) [2, 3, 4]. However, certain (capacitive) types of impedance, corresponding to the step like wakefunctions, have been known for some time [5, 2] to induce the opposite effect: bunch shortening. This wakefield focusing effect is documented numerically and experimentally in LEP [3] . The comprehensive theory was developed more recently by Burov [6, 7], proving as well the coherent stability of the wakefield-focused bunches. Wakefield focusing is achieved through the short-range part of the wakefield; it is a broad-band focusing, as distinct from narrow-band focusing by the RF cavities.

The first original proposal for a special device that would produce a step-like wakefield and allow compression of the bunch several times, was to use a dielectric axial-symmetric canal with thick wakefield-absorbing walls [8]. No adverse single- or coupled-bunch instabilities are induced in this scheme, but only at the price of sizable power dissipation that can be prohibitive in some applications. A method of circumventing this problem was

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proposed in [9] by modifying the canal with wakefield-absorbing walls into a closed-ends nonabsorbing waveguide that presents a high-quality cavity. The total energy losses can be reduced dramatically, but strong coupled-bunch instabilities appear due to the strong high-order modes (HOM's) of the cavity that are close in frequency to the fundamental. In this report, we propose the ultimate compromise between the two ideas which results in a moderately lossy wakefield focusing device: an open-ended waveguide, with the wakefield energy absorbed at the ends. The device presents itself a long pipe with a thin low-loss dielectric lining inside. At both ends of the structure, the lossy dielectric is used to provide for the absorption of the longitudinally propagating waves. The idea of using the canal as a waveguide rather than a cavity was suggested to us by A. V. Novokhatski [10]. The thrust of this idea is the removal of the neighboring HOM's and the associated coupled-bunch instabilities.

Wake functions for the dielectric canal have been found in Refs.8 and 11-13. Assuming external and internal radii of the dielectric layer to be a and $b < a$, the length of the waveguide L and the dielectric constant ε, the longitudinal wake function can be presented for the thin-lining case $\tilde{\delta} = 2\varepsilon(1 - b/a) \ll 1$ as

$$W_{||}(z) = A\left(\cos(k_0 z) + \frac{2\tilde{\delta}}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(k_n z)}{n^2}\right) \tag{1}$$

with amplitude $A = 4L/b^2$, higher harmonic wavenumbers $k_n = \pi n k_0/\sqrt{\tilde{\delta}}$, and the lowest harmonic wavenumber

$$k_0 = \frac{1}{a}\sqrt{\frac{2\varepsilon}{(\varepsilon - 1)(1 - b/a)}}. \tag{2}$$

The transverse wake function in the same leading approximation in $\tilde{\delta}$ is

$$W_{\perp}(z) = \frac{2A}{k_0 b^2}\sin(k_{\perp} z) \tag{3}$$

where the transverse wavenumber in the leading-order approximation equals the longitudinal one: $k_{\perp} = k_0$.

The imaginary part of the dielectric constant ε'' translates into the decay of the wake functions, described by the imaginary part of the wavenumber k''. The quality factor of the fundamental mode $Q = k/(2k'')$ is found to be several times larger than that of the higher-order modes:

$$Q_0 = \frac{k_0}{2(\partial k_0/\partial\varepsilon)\varepsilon''} = \frac{\varepsilon^2}{\varepsilon''}, \quad Q_n = \frac{k_n}{2(\partial k_n/\partial\varepsilon)\varepsilon''} = \frac{\varepsilon - 1}{\varepsilon^2}Q_0. \tag{4}$$

The quality factor of the transverse wake mode equals that of the fundamental longitudinal mode: $Q_{\perp} = Q_0$.

The previous calculation assumed an infinitely long waveguide. For a finite but long waveguide with absorbing ends the only significant change is the reduction of the quality factors. Indeed, the field energy moves along the waveguide with group velocity $u = d\omega/dk$, which results in the decrement rate $\Delta k'' = u/(cL)$ due to absorption at the exit of the waveguide. Using the dispersion relation of the waveguide

$$\omega^2/c^2 = (q_0^2 + k^2)/\varepsilon, \tag{5}$$

with $q_0 = k_0\sqrt{\varepsilon - 1}$, taking the derivative of Eq. (5) at $k = k_0$, and substituting $\omega = ck$, the group velocity is found to be $u = c/\varepsilon$. The quality factor then is $Q_0 = k_0/\Delta k'' = \epsilon k_0 L$. The "compromise" regime that allows reduction of the power losses is when the wakefield decay distance is much larger than the bunch spacing, but small enough to provide resonably low coupled-bunch increments.

The external RF potential well is distorted by the wakefields. Assuming thermal equilibrium of the bunch particles, the self-consistent bunch density $\rho(z)$ can be shown to satisfy a Haissinski equation [14]:

$$\frac{d\rho(z)}{dz} = \rho(z)[z + F(z)], \tag{6}$$

where the total wakefield force F is

$$F(z) = -\frac{Ne^2}{\kappa}\int_{-\infty}^{z} dz' W(z - z')\rho(z') - \frac{Ne^2}{\kappa}\sum_{n=1}^{\infty}\int_{-\infty}^{\infty} dz' W(z + nl - z')\rho(z'), \tag{7}$$

the distances are measured in units of unperturbed bunch length $\sigma_0 = 1$, N is the number of particles in the bunch, κ is the RF focusing rigidity (proportional to the RF voltage and the wavenumber), and l is the bunch spacing. The density ρ is normalized: $\int dz\rho(z) = 1$.

The case of low quality factor and step-like wake functions was analyzed in [6, 7, 8], identifying this regime as the bunch self-focusing. Low quality factor here means that the wakefields decay within the distance between the bunches, and step-like means being almost constant for distances larger than the bunch length, so that the Heavyside step-function $\theta(z)$ is a valid approximation, $W(z) \propto \theta(z)$. The bunches were shown to be stable in this regime for arbitrarily strong wakefields, and the bunch length reduction to be limited only by the energy losses. The average value of the wake force over the bunch in a stationary configuration represents the average energy loss per bunch. Therefore, in this regime an unavoidable energy loss is incident to the useful focusing effect. A different alternative to high quality factor step-like single-mode wakefield focusing was studied numerically in [9], and the possibility of greatly reduced energy losses with the same focusing effect was demonstrated. In order to obtain some analytical results for this regime, consider a single-mode wakefield $W(z) = A\cos(kz)\exp(-k''z)$ with step-like $k\sigma \ll 1$, and low quality factor conditions $k''l \ll 1$ both satisfied (here σ is the r.m.s. bunch length). The energy loss per bunch per turn around the ring ,E, is obtained as the average value of the wakefield force (7) over the bunch distribution [15]:

$$E = \int_{-\infty}^{\infty} dz\rho(z)\int_{-\infty}^{z} dz'\rho(z')F(z - z') = A/2 + A\,\mathrm{Re}\left\{\frac{1}{\exp(-i\psi + \psi'') - 1}\right\} \tag{8}$$

where the first term is the same-bunch contribution, in agreement with the beam-loading theorem, while the second term comes from the wakefields induced by the previous crossings. The phases ψ and ψ'' are defined as $\psi = kl$, $\psi'' = k''l$. One can see that in the anti-resonance regime $\psi = \pi(2m+1)$ the energy losses are suppressed: $E = A\psi''/4$, with the suppression factor $\psi''/2$ (see also [9]). The wakefield force $F(z)$ (7) in this case can be presented near the center of the bunch $|z| \ll 1/k''$ in the form

$$F(z) = -w \int_{-\infty}^{z} dz' \cos(k(z-z'))\rho(z') + (w/2)\cos(kz) \int_{-\infty}^{\infty} dz' \cos(kz')\rho(z'), \qquad (9)$$

where $w = Ne^2 A/\kappa$.

The leading-order estimation of the bunch compression effect in the strong focusing regime $w/2 \gg 1$ can be obtained [6] from Eq. (9) in the limit of zero bunch length:

$$F(z) = -(w/2)\text{sign}(z). \qquad (10)$$

Substituting the force (10) in the Haissinski equation (6) one obtains the distribution

$$\rho(z) = \frac{w}{8}\frac{1}{\cosh^2(wz/4)} \qquad (11)$$

which corresponds to a bunch length $\sigma = 3.6/w$. It is possible as well to derive a single rational-function approximation that fits the bunch length for arbitrary w [16].

An example of the bunch distribution compression obtained by the numerical solution of the Haissinski equation (7) with the force (9) is shown in Fig. 1. The wake focusing parameter is $w = 6$, the wavenumber $k = 0.5$ and the bunch length is reduced by half.

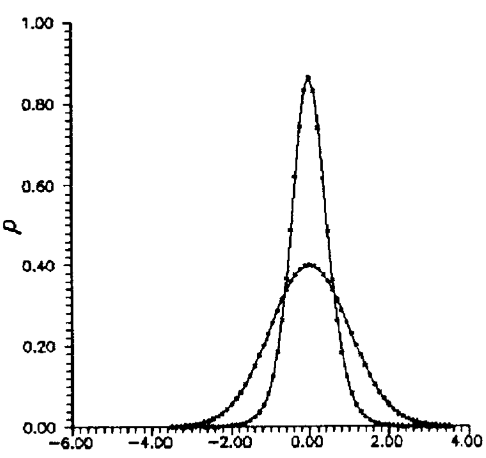

Fig. 1 Wakefield-compressed distribution with high quality factor single-mode wakefield with parameters $w = 6$, $k = 0.5$ (higher and narrower curve). The lower and wider curve is the unperturbed Gaussian distribution.

The focusing effect deteriorates for larger values of k. The bunch length dependence on the focusing parameter from the numerical solution of the Haissinski equation is shown for various wavenumbers in Fig. 2. It can be concluded that the difference between $k = 0$ and $k = 0.5$ is rather small, while the case $k = 1.0$ (the highest curve) is the boundary of the self-focusing area.

For any value of $k < 1$, the function $\sigma(w)$ goes through a minimum and starts growing again (not shown in Fig.2). This is due to the additional minima in the total potential well that appear for sufficiently large w. In this case the bunch splits into subbunches [5, 6, 7]. No additional minima can be shown to exist for $wk \leq 5.9$, so that the minimum of $\sigma(w)$ occurs approximately at $w = 5.9/k$.

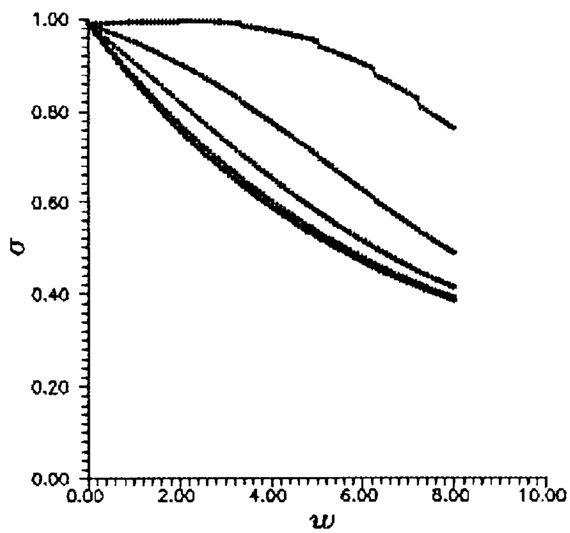

Fig. 2 Bunch length σ as a function of the wake focusing parameter w for various wavenumbers. The lowest curve corresponds to $k = 0$ while the other curves, monotonically rising with k, are for $k = 0.25,\ 0.5, 0.75$ and 1.

How should the parameters of the waveguide be chosen in order to provide a sufficiently large w and wavelength $1/k$? For the same values of w and k, the thinner dielectric layer requires a larger radius and length of the waveguide. To keep the length minimum, a larger layer thickness should be chosen. One is limited, however, by the effect of higher-order wakefield harmonics in eq.(1) with $n = 1, 2, \ldots$. These harmonics cause an additional power loss that can not be recuperated. The upper limit on the layer thickness that is still tolerable can be obtained by equating the higher-order mode losses with the fundamental mode loss.

The ratio of the energy losses due to the $n = 1$ wakefield harmonic to the uncompensated (low quality factor) single-bunch losses in the main harmonic $n = 0$, is found from the standard parasitic loss theory (see, e.g., [15]) as

$$\frac{\Delta E_1}{\Delta E_0} = \frac{2\tilde{\delta}}{\pi^2}\rho_1^2, \quad \rho_1 = \int_{-\infty}^{\infty} \cos(k_1 z)\rho(z)dz. \tag{12}$$

The Fourier image of the bunch density ρ_1 and the losses are reduced exponentially strongly when the wavenumber becomes significantly larger than the inverse bunch length, $k_1 \propto k_0/\sqrt{\tilde{\delta}} > 1/\sigma$. More detailed estimates of losses are presented in [16].

The paramount issue in wakefield focusing is providing for coherent stability. The growth rate $\Lambda_{||}$ of the longitudinal coupled-bunch mode μ can be expressed (see, e.g. [15]) as

$$\Lambda_{||} = \frac{\alpha M N r_0 c}{4\pi\gamma\nu_s T_0} \mathrm{Re} \sum_{p>0} (pM\omega_0 - \mu\omega_0) Z(pM\omega_0 - \mu\omega_0) - (pM\omega_0 + \mu\omega_0) Z(pM\omega_0 + \mu\omega_0), \tag{13}$$

where α is the momentum compaction, M is the number of bunches, ν_s is the synchrotron tune, $T_0 = 2\pi/\omega_0$ is the revolution period and $Z(\omega)$ is the impedance. Using the impedance of the fundamental mode tuned in antiresonance, one finds that the fastest growing mode occurs for $|\mu\omega_0 - M\omega_0/2| = ck''$ when $\sum_{p>0}\{...\} = A/2$. This worst-case growth rate can be expressed in terms of the wake focusing parameter w (9):

$$\Lambda_{||} = \frac{w\sigma_0}{4l}\omega_s = \frac{w}{4}\frac{\alpha c\sigma_\epsilon}{l}, \tag{14}$$

where ω_s is the synchrotron frequency, σ_ϵ is the r. m. s. width of the energy distribution of the bunch particles, $\sigma_0 = \alpha c\sigma_\epsilon/\omega_s$ is the natural bunch length, and l is the bunch spacing.

The higher-order harmonics of the wake function can drive microwave instabilities when the wavelengths of these modes are significantly shorter than the bunch length. The stability condition for the microwave mode with a wavenumber k can be expressed, using the standard estimates (see, e.g. [15]), as:

$$\frac{2\tilde{\delta}}{\pi^2}\frac{w\sigma_0}{l(k\sigma_0)^2} Q_k \leq 1. \tag{15}$$

Thus the quality factors of the higher order harmonics Q_k should not be too high. For the KEK B-factory parameters (see below), one obtains the requirement $Q_k \leq 800$. According to the previous analysis, Eq. 4, the quality factor of the fundamental mode is $\varepsilon^2/(\varepsilon - 1) \simeq 4$ to 5 times larger than the quality factors of the higher-order harmonics; thus the microwave stability imposes an upper limit on the quality factor of the fundamental mode. For the KEKB case, one obtains $Q_0 \leq 3 \cdot 10^3$, which corresponds to the the lower limit on the losses suppression factor $\psi'' \geq 0.01$.

Consider now the transverse coupled-bunch modes. Their growth rates [15] are

$$\Lambda_\perp = \frac{MNr_0\beta}{2\gamma T_0^2} \mathrm{Re} \sum_p Z_\perp(pM\omega_0 + \nu_b\omega_0 - \mu\omega_0) \tag{16}$$

where ν_b is the betatron tune, and $\beta \approx R/\nu_b$ is the average beta-function at the location of the waveguide. The transverse wake wavenumber is slightly detuned from the longitudinal one $|k_\perp - k_0| \geq k''$. This detuning has a detrimental effect on the transverse stability and the fastest-growing mode growth rate can be found to be

$$\Lambda_\perp = \frac{MQ_0\sigma_\epsilon\nu_s w}{2\pi\nu_b(k_0b)^2}\omega_0. \tag{17}$$

This instability can be Landau damped if the betatron tune spread is large enough. The betatron tune spread in electron-positron B-factories is due mostly to the beam-beam effects and it results in a Landau damping rate $\lambda_\perp = 0.16\xi\omega_0$ [17], where ξ is the beam-beam tune shift. Thus, the transverse coupled-bunch stability condition can be expressed in terms of the tune shift:

$$\frac{\Lambda_\perp}{\lambda_\perp} = \frac{MQ_0\sigma_\epsilon\nu_s w}{(k_0b)^2\nu_b\xi} \leq 1. \tag{18}$$

In the case where the stability condition (18) is not satisfied, or for applications in synchrotron light sources without any beam-beam interaction, one can use a special modification for waveguide shape (longitudinal radius variation) which effectively reduces the transverse impedance mode quality factor without any change in the longitudinal mode quality factor [16].

To illustrate our concept, we describe now a specific proposal for a wakefield focusing scheme for the Low Energy Ring of the future KEK B-factory [18]. Major obstacles to achieving shorter bunches in the B-factories are the coupled-bunch instabilities and bunch lengthening due to the broad-band impedance, both of which get worse when increasing the number of RF cavities ([18, 1]). We propose the use of the dielectric waveguide as the prevalent means of the longitudinal focusing, with the RF cavities serving mostly as the power sources for the beam. The benefit of our scheme is either reduction of the number of cavities for the same bunch length (scenario I) or reduction of the bunch length for the same number of cavities (scenario II), depending on which of these options is more appealing in view of various technical constraints.

Table I. Parameters for three scenarios.

Scenario #		I	II	III
Number of RF cavities		8	16	16
RF voltage (MV)	V	4	8	8
Bunch length (mm)	σ	4	2.7	4.
Waveguide length (m)	L	10.4	10.4	0.
Radius of the canal(cm)	a	5.6	4.	
Dielectric layer thickness (cm)	$d = a - b$	0.7	0.5	
Waveguide power loss (kW)	P_d	80	160	0.
Waveguide wakefield frequency (GHz)	$ck_0/(2\pi)$	4.1	5.6	
Total wavevegude voltage (MV)	ANe^2	1.4	2.7	0.
RF power loss in cavities (MW)	P_c	1.15	2.3	2.3
Long. instab. growth time (ms)	$\Lambda_\parallel^{-1}$	25	25	27

In Table I we present the parameters for the three scenarios, including the original parameters for comparison. Each of the ARES RF cavities produces 0.5 MV voltage and dissipates 140 kW of power [19]. In scenario I, we reduce by half the number of RF cavities, presently planned at 16, and save about 1.1 MW of power. In scenario II, we reduce the bunch length by a factor 1.5, with only about 8% total power loss increase. Note that significant bunch shortening is accomplished with only 2.1 MV of the secondary (waveguide) RF voltage, which illustrates the efficiency of the wakefield focusing. The number of particles per bunch in all cases is $N = 3.3 \cdot 10^{10}$, momentum compaction is $\alpha = 1.7 \cdot 10^{-4}$, dielectric constant is $\varepsilon = 3$ and dielectric loss factor is $\varepsilon''/\varepsilon = 0.01$. For comparison, we present the parameters for scenario III, which is the presently planned KEK design with 16 RF cavities.

The lowest harmonic of the wakefield drives the instability with a coupled-bunch mode index μ that is different from the index of coupled-bunch modes in the present KEK plan (scenario III) due to the fundamental mode of the main RF cavities. Moreover, the stability criterion is somewhat unusual since the the whole range of the incoherent tunes can be easily shown to be shifted beyond the coherent tune (that does not change much by comparison). As a result, there is no Landau damping for the wakefield-driven dipole coupled-bunch modes even though the incoherent tune spread is fairly large $\delta\nu_s/\nu_s \approx 0.3$ (about three times as large as in the presently planned design). In our proposal, we imposed the restriction that the growth times of these new modes are about the same as those of scenario III, which are low enough to be stabilized by the radiative damping due to the synchrotron radiation. Thus, there is no deterioration of coupled-bunch stability. It may turn out that the new coupled-bunch modes are easily damped by a narrowband feedback system. In this case, still shorter bunches can be be obtained by using a longer waveguide in scenario II.

To provide for stability of the transverse coupled-bunch modes we opted to lower the quality factor of the wakefields, both transverse and longitudinal, by utilizing the lossy dielectric $\varepsilon''/\varepsilon = 0.01$ all along the waveguide. This way, it is not necessary to resort to technically difficult waveguide radius shaping (see [16]) for reduction of the transverse impedance quality factor. The overall power losses incurred are still at a negligible level (see Table I). Another consequence of this scheme is that the waveguide can be cut into three equal pieces and installed separately without any deterioration of quality factors. On both ends of the structure the absorbing units are placed that capture the waves without any reflections. These units can present themselves pieces of the same waveguide of about $0.5m$ length and same radius, with the dielectric with the same ε but high losses $\varepsilon'' \approx 0.2$. For the single- piece waveguide, 75% of the power loss is dissipated in the bulk of dielectric and the rest 25% in the absorbing ends, so we do not expect the cooling of the structure to be a problem.

The transverse multi-bunch modes, driven by the waveguide impedance, are estimated in both scenarios to be stable when beam-beam tune spread is present. The instability threshold for number of particles per bunch for more unstable scenario II is close to the threshold for the current design. When beams are not in collision, transverse instability can be damped by a narrowband transverse feedback system. Another alternative is to "turn off" the wakefield at this time by mechanically inserting a shielding pipe inside the waveguide.

In principle, the wakefield focusing technology may be very useful for the synchrotron light sources. A significant difficulty though is that these machines are often operated in the regime of the turbulent bunch lengthening, and not of the static potential well distortion. The effect of the parasitic broad-band impedances in the wakefield focusing was not considered in the present study as it is not very strong for the B-factories (which are assumed to be operated under the turbulent bunch lengthening threshold). More studies are needed of this effect to prove more convincingly the practicality of the wakefield focusing, but we feel that the basic notion is proved viable.

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